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Improved Al-Marhoun Correlation for Bubble Point Pressure of Libyan Crude Oils

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ABSTRACT

Accurate estimation of crude oil bubble point pressure plays a vital role in many petroleum engineering calculations, such as reserve estimation, material balance, reservoir simulation, production equipment design, and optimization of well performance. The P_b can be measured in the pressure-volume-temperature (PVT) experiments. Nonetheless, the PVT measurements have limitations, such as being costly and time-consuming. In this paper, the Al-Marhoun correlation for bubble point pressure for black oil reservoirs is improved using the linearization of non-linear regression techniques to fit Libyan crudes accurately. A total of 62 PVT data reports taken from various Libyan oil fields were used in the study. The PVT data consist of an oil gravity range of 24.7 ºAPI to 46.8 ºAPI and bubble point pressures of 123 psig to 6100 psig. Statistical error analyses and graphical methods were used to evaluate the original and modified Al-Marhoun correlations. The results showed that the improved Al-Marhoun correlation exhibits significantly lower average absolute error and deviation than the published ones. The correlation coefficient of R^2 of the improved Al-Marhoun correlation is 96.70%, and the average percent error (APE) is 0.70% with a standard deviation (SD) of 14.70.

Keywords: bubble point pressure; PVT, non-linear regression analysis, Statistical error analyses, Libyan crudes

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تحسين علاقة المرهون لضغط نقطة الفقاعة للنفط الخام الليبي

صـــالح الرويـنـــي
قسم هندسة النفط، كلية الهندسة، جامعة طرابلس، طرابلس، ليبيا

ملخـــــص البحــــث

يلعب التقدير الدقيق لضغط نقطة فقاعة النفط الخام دورًا حيويًا في العديد من حسابات هندسة النفط، مثل تقدير الاحتياطي، وتوازن المواد، ومحاكاة المكامن، وتصميم معدات الإنتاج، وتحسين أداء البئر. يمكن قياس ضغط نقطة فقاعة p_b في تجارب الضغط والحجم ودرجة الحرارة (ضغط وحجم ودرجة حرارة). ومع ذلك، فإن قياسات الضغط والحجم ودرجة الحرارة لها حدود، مثل كونها مكلفة وتستغرق وقتًا طويلاً. في هذا البحث تم تحسين علاقة المرهون لضغط نقطة الفقاعة لمكامن النفط الأسود تقنيات تحويل الانحدار الغير خطي إلى خطي لتناسب الخام الليبي بدقة. تم استخدام ما مجموعه 62

تقرير بيانات الضغط والحجم ودرجة الحرارة مأخوذة من مختلف حقول النفط الليبية في الدراسة. تتكون بيانات الضغط والحجم ودرجة الحرارة من نطاق الكثافة النوعية حسب معهد النفط الأمريكي °API من 24.7 إلى 46.8 وضغوط نقطة الفقاعة من 123 رطل لكل بوصة مربعة إلى 6100 رطل لكل بوصة مربعة. تم استخدام تحليل الأخطاء الإحصائية والأساليب الرسومية لتقييم علاقة المرهون الأصلية والمحسنة. أظهرت النتائج أن علاقة المرهون المحسنة اعطت متوسط خطأ وانحراف مطلق أقل بكثير من تلك المنشورة. وبلغ معامل الارتباط R^2 لعلاقة المرهون المحسنة بـ 96.70%، ومتوسط نسبة الخطأ (APE) بـ 0.70% مع انحراف معياري (SD) وقدره 14.70.

الكلمات المفتاحية: ضغط نقطة الفقاعة **PVT**، تحليل الانحدار غير الخطي، تحليل الأخطاء الإحصائية، الخامات الليبية.

1. Introduction

The bubble-point pressure (P_b) of a hydrocarbon system is defined as the highest pressure at which a bubble of gas is first liberated from the oil [1]. For appropriate material balance, reservoir, and petroleum production calculations, an accurate reservoir P_b is essential. As a result, an inaccurate estimate of bubble point pressure will undoubtedly propagate errors in other oil PVT parameters. Ideally, laboratory tests on collected bottom-hole reservoir fluid samples or mixed surface samples are the most precise technique to evaluate PVT parameters, including bubble point pressure. However, in practice, this option is not always available due to a variety of factors, including insufficient or contaminated samples, the high expense of the accompanying experiments, or the fact that these tests are often carried out for specific pressure and temperature ranges. Therefore, empirical correlations should be employed in the absence of lab measurements [2].

The reservoir's pressure would drop even further with production, yielding substantial amounts of gas that might predominate the reservoir's multiphase liquid flow. A high gas-oil ratio (GOR) at producing wells is anticipated once there is sufficient gas production. Knowing the bubble point pressure is essential for making important judgments in reservoir engineering. To keep reservoir pressure above the bubble point and prevent gas development and its eventual dominance in oil production, early pressure management operations may be required. Reinjection of the produced gas may be required to keep reservoir pressure at the ideal level if the initial reservoir pressure is below the bubble point pressure and there is a gas cap present [3].

Several graphical and mathematical correlations for determining P_b have been proposed during the last four decades. These correlations are essentially based on the assumption that the bubble-point pressure is a strong function of gas solubility R_s, gas gravity γ_g, oil gravity °API, and temperature T, or [4].

$$P_b = f(R_S, \gamma_g, °API, T)$$

One of the most well-known PVT correlations in the oil and gas sector is the Al-Marhoun correlation. The majority of well-known software, like PROSPER, MBAL, PIPESIM, etc., included this correlation because of its accuracy and reliability. The objective of this study is to evaluate its applicability for some crude oils collected from different Libyan oilfields and then modify the correlation coefficients to fit Libyan crude oils with less error. In this study, linearization of non-linear regression analysis is used to tune correlation coefficients to give more accurate predictions.

2. Related Work

Standing [5] and Lasater [6] used graphical methods to predict the P_b. Standing utilized more than 100 datasets from US crude oil to develop a correlation for the prediction of P_b. Lasater published an equation to predict P_b using 158 datasets from Canada and the United States. In the 1980s to 1990s, some researchers used linear and nonlinear regressions to determine the P_b. Vasquez and Beggs employed linear regression analysis to determine the P_b, operating on 6004 datasets from different fields [7]. Glaso [8] used linear and nonlinear regressions to determine the P_b and stated that his correlation has a standard deviation of 6.98. Other authors used linear and nonlinear regression to predict P_b using data from different parts of the globe. Al-Marhoun [9], Kartoatmodjo and Schmidt [10], Dokla and Osman [11], Petrosky and Farshad [12], Macary and El-Batanoney [13], De Ghetto *et al*. [14], and Hanafy *et al*. [15] utilized multiple regressions to predict the P_b. In the literature, there are several other correlations that have been used successfully around the world [16-21]. Table 1 presents a review of the well-known bubble-point pressure correlations published in the literature and shows authors, published year, and sample origin.

Table 1. Literature bubble-point pressure correlations

Authors	Published year	Sample origin	Reference
Standing	1947	California	[5]
Lasater	1958	Canada and USA	[6]
Vazquez and Beggs	1980	Worldwide	[7]
Glaso	1980	North sea	[8]
A1-Marhoun	1988	Middle-East	[9]
Labedi	1990	Libya, Nigeria and Angola	[16]
Dokla and Osman	1992	U.A.E.	[11]
Macary and El Batanony	1993	Gulf of Suez	[13]
Omar and Todd	1993	Malaysia	[17]
Petroskv and Farshad	1993	Gulf of Mexico	[12]
De Ghetto *et al.*	1994	Mediterranean Basin, Africa, the Persian Gulf and the North Sea	[14]
Kartaotmodjo and Schmidt	1994	Worldwide	[10]
Almehaideb	1997	U.A.E.	[18]
Hanafy *et al.*	1997	Egypt	[15]
Al-Shannuasi	1999	Worldwide	[19]
Velarde *et al.*	1999	Worldwide	[20]
Hemmati and Khanat	2007	Iran	[21]
Fattah and Lashin	2018	Egypt	[22]

2.1 Original Al-Marhoun Correlation

Al-Marhoun [9] presented a paper on the correlations of bubble-point pressure and total formation volume factor for Middle Eastern crude oils as a function of reservoir temperature, gas gravity, oil gravity, and solution gas/oil ratio. The correlation was computed over 69 bottom-hole samples, with 160 data points for the bubble-point pressures. Table 2 shows the ranges of the data used to develop the correlation. A non-linear multiple regression analysis was used to develop the following correlation, where T is described in ºR:

$$P_b = a_1 R_s^{a2} \gamma_g^{a3} \gamma_o^{a4} T^{a5} \quad (1)$$

where T is the temperature in °R, γ_o is the stock-tank oil specific gravity (water = 1), γ_g is the gas specific gravity (air = 1), R_s is the gas solubility, scf/STB, and a_1–a_5 are coefficients of the correlation having the following values:

$a_1 = 0.00538$, $a_2 = 0.71508$, $a_3 = -1.87784$, $a_4 = 3.14370$ and $a_5 = 1.32657$

Table 2. Data Parameters and Ranges for Al-Marhoun [8]

PVT Property	Range	Unit
Bubble point pressure	130 to 3573	psia
Pressure	20 to 3573	psia
Bubble point oil FVF	1.032 to 1.997	RB/STB
Total FVF below P_b	1.032 to 6.982	RB/STB
Solution GOB,	26 to 1602	Scf/STB
Reservoir Temperature	74 to 240	°F
Average gas relative density (air.= 1)	0.752 to 1.367	dimensionless
Stock-tank oil gravity	19.40 to 44.6	°API
CO_2 In surface gases	0.00 to 16.38	mol%
Nitrogen n surface gases	0.00 to 3.89	mol%
H_2S in surface gases	0.00 to 16.13	mol%

3. Data Description

Sixty-two laboratory PVT reports were collected from various Libyan oil reservoirs of different chemical compositions. Table 3 represents a description of the data utilized in this study within wide ranges of pressure, temperature, solution gas-oil ratio, oil gravity, and oil viscosity.

Table 3. Description of the PVT data used in this study

Property	Unit	Min	Max
Bubbel point pressure (P_b)	psia	123	6100
Temperature (T)	°F	132	300
Solution GOR at P_b (R_{sb})	scf/STB	28	2156
Stock-Tank Oil Gravity (γ_{API})	°API	24.7	46.8
Specific Gas Gravity (γ_g)	Air =1	0.701	1.462
Dead oil viscosity (μ_{od})	cp	0.774	5.036
Saturated oil viscosity (μ_{ob})	cp	0.200	3.811
UnderSaturated oil viscosity (μ_o)	cp	0.123	6.584
Oil formation volume factor (B_o)	bbl/STB	1.035	2.220

4. Methodology

4.1 Non-linear Regression Technique

Nonlinear systems are ubiquitous in engineering, physics, and mathematics. They exhibit complex behavior that can be difficult to analyze, making it challenging to design control systems or predict outcomes. One way to overcome the complexity of nonlinear systems is through linearization. Al-Marhoun is a power model; therefore, transformations can be used to express the data in a form that is compatible with linear regression. The linearization procedure is summarized in the following three steps:

1. Transform the dependent and/or independent data values.
2. Apply linear least-squares regression. One strategy for fitting a "best" line through the data would be to minimize the sum of the squares of the residuals between measured and simulated quantities.

$$Minimize \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(P_{b,measured} - P_{b,claculated}\right)_i^2 \qquad (2)$$

   where $P_{b,measured}$ is the measured bubble-point pressure, psia, $P_{b,calculated}$ is the calculated bubble-point pressure, psia, n is the total number of points

3. Inverse transform the determined coefficients back to those that define the nonlinear functional relationship.

In this study, a Microsoft Excel spreadsheet was built to execute the linearization of non-linear regression analysis. The coefficients for the bubble-point pressure correlation developed by Al-Marhoun [9] were regressed through the experimentally obtained data to improve the estimation. The old and new coefficients of the Al-Marhoun correlation are listed in Table 4.

Table 4. Original and modified Al-Marhoun coefficients

Coefficient	Original	Modified
a_1	0.00538	0.0000621
a_2	0.71508	0.7960520
a_3	−1.87784	-0.7072300
a_4	3.14370	5.9700060
a_5	1.32657	2.0471520

4.2 Performance Evaluation Tools

Both statistical and graphical tools have been utilized simultaneously to evaluate the performance of the studied correlations in terms of their accuracy.

4.2.1 Statistical Error Analysis

To evaluate the performance of the studied model in predicting the bubble point pressure, various statistical indicators were utilized, such as Percent Error (PE), Average Percent Error (APE), Average Absolute Percent Error (AAPE), Minimum Error (MIN), maximum error (MAX), standard deviation (SD), and the coefficient of determination (R^2). These indicators are presented by Equations (3) – (9) as follows:

$$\text{Percent Error (PE)} = \frac{p_{b_{exp}} - p_{b_{cal}}}{p_{b_{mes}}} \tag{3}$$

$$\text{Average Percent Error (APE)} = \frac{1}{n}\sum_{i=1}^{n} PE_i \tag{4}$$

$$\text{Average Absolute Percent Error (AAPE)} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\left(p_{b_{exp}} - p_{b_{cal}}\right)_i}{\left(p_{b_{mes}}\right)_i}\right| x\ 100 \tag{5}$$

$$\text{Minimum Error} = \min_{i=1}^{n}\left[\left|\frac{\left(p_{b_{exp}} - p_{b_{cal}}\right)_i}{\left(p_{b_{mes}}\right)_i}\right|\right] \tag{6}$$

$$\text{Maximum Error} = \max_{i=1}^{n}\left[\left|\frac{\left(p_{b_{exp}} - p_{b_{cal}}\right)_i}{\left(p_{b_{mes}}\right)_i}\right|\right] \tag{7}$$

$$\text{Standard Deviation (SD)} = \sqrt{\frac{\sum_{i=1}^{n}(PE_i - APE_i)^2}{n-1}} \tag{8}$$

$$\text{Coefficient of Correlation } (R^2) = 1 - \frac{\sum_{i=1}^{i=n}\left[\left(p_{b_{exp}} - p_{b_{cal}}\right)_i\right]^2}{\sum_{i=1}^{i=n}\left[\left(p_{b_{exp}} - \overline{p_b}\right)_i\right]^2} \quad (9)$$

where

$$\overline{p}_b = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\left(p_{b_{exp}}\right)_i,$$

where $p_{b_{exp}}$ is the experimental pubble-point pressure, psia, $p_{b_{cal}}$ is the calculated pubble-point pressure, psia, n is the number of points

4.2.2 Cross Plot

To visualize the accuracy and performance of a correlation, all the calculated values are plotted versus the experimental (measured) values, and thus a crossplot is formed. On the crossplot, a 45° straight line is drawn with estimated values equaling experimental values. The closer the plotted data points are to this line, the better the correlation.

5. Results and Discussion

Al-Marhoun [9] provides Equation 1 for estimating the bubble-point pressure of Middle East crude oil samples. A total of 62 PVT lab reports were collected, screened and utilized for our study. Al-Marhoun correlations for bubble point pressure coefficients were recalculated using linearization of non-linear regression analysis to reduce the error and improve the performance of the correlations. Based on datasets from different Libyan fields, new coefficients were generated as seen in Table 4.

Based on statistical analysis, the Al-Marhoun correlation has been much improved after modifying its coefficients. Table 5 provides the statistical performance indicators of each correlation in terms of APE, AAPE, SD, R^2, MIN, and MAX values. In observing the results, it can be noted that modified Al-Marhoun has much improved and gave the lowest error compared to the original correlation, with an APE of 0.7, an AAPE of 10 %, a SD of 14.66%, and an R^2 value of 96.7%. Figure 1 and Figure 2 show the crossplot for both the original and modified Al-Marhoun. Most of the data points of the modified correlation fall very close to perfect correlation of the 45° line.

Table 5. Statistical Accuracy of P_b correlations before and after Optimization

Correlation	**APE**	**AAPE**	**SD**	**R^2 %**	**MIN**	**MAX**
Original Marhoun	17.1	23.08	22.68	82.0	0.04	54.92
Modified Marhoun	0.7	10.00	14.66	96.7	0.01	33.38

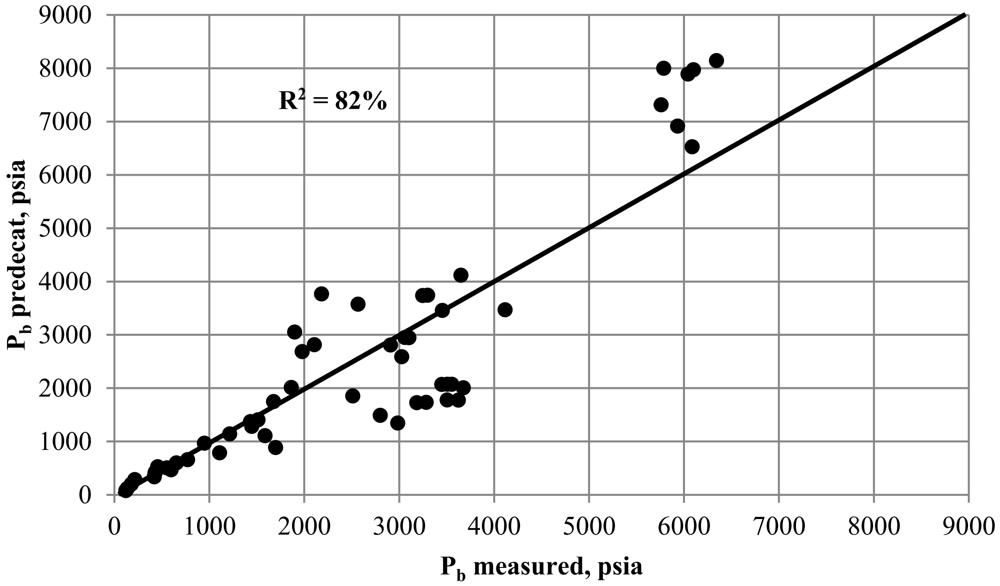

Figure 1. Origenal Al-Marhoun correlation.

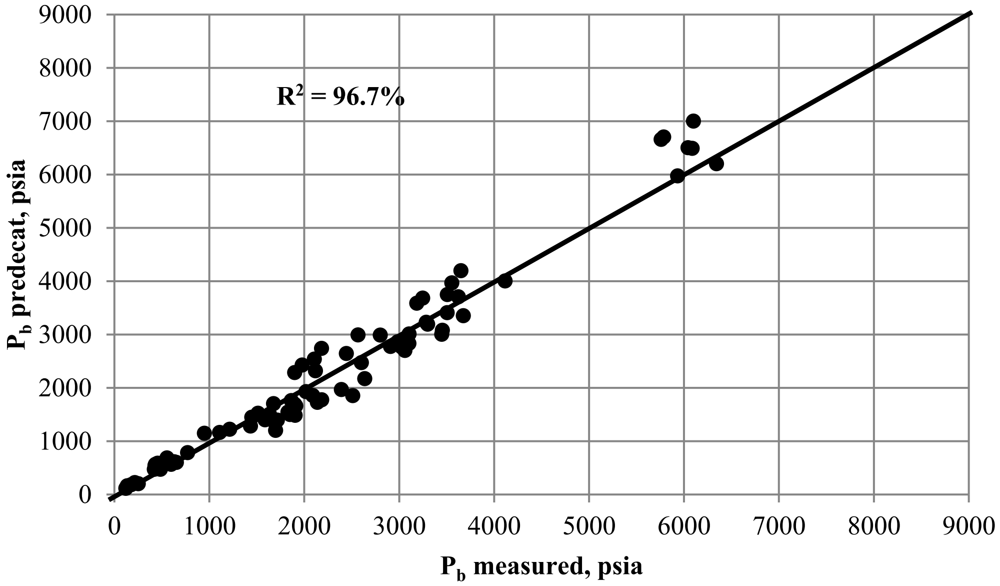

Figure 2. Modified Al-Marhoun correlation.

6. Conclusions

Based on the dataset analyzed in this study, the following conclusions were drawn:

1. 62 PVT reports were collected from different Libyan crudes. The dataset covers a wide range of fluid properties for black oil and is thoroughly reviewed for overall quality before using the studied correlation.
2. A set of new correlation coefficients is successfully generated using the linearization of non-linear regression analysis to predict better bubble-point pressure.

3. Based on statistical accuracy, the improved Al-Marhoun correlation for bubble-point pressure demonstrates a significant improvement compared to the original correlations.
4. Higher accuracy with an R^2 of 96.65% is obtained when the correlation is tuned to the regional data, and the average absolute error is 10% with a standard deviation of 14.66.

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