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Calculating Deformation Parameters and the Electric Transition Probability of the Even-Even Nuclei for $^{54-62}$Ni Isotopes

Mariam O Madi[a*], Fozia O Madi[b], and Ayad E Ezwam[c]
[a, c]Department of Physics, Faculty of Science, University of Tripoli, Tripoli, Libya
[b]Faculty of Civil Aviation, Misurata, Libya
**Corresponding author:*

E-mail address: ma.madi@uot.edu.ly

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A B S T R A C T
The current work focused on studying the shape and properties of even-even nuclei form that have the mass number less than 100 (A < 100) for ($^{54-62}_{28}$Ni) isotopes by using deformed shell model equations. The values of the deformation parameter (β_2) and intrinsic electric quadrupole moments (Q_0) derived from calculating the reduced electric quadrupole transition probability $B(E2)\uparrow$ from the ground 0^+ to the first-exited 2^+ state. The calculated values for $B(E2)\uparrow$ and β_2 compared with the experimental data obtained from the National Nuclear Data Center (NNDC), there was little change in the result. The relationship between their calculated values for ($B(E2)\uparrow$, β_2 , Q_0) and neutrons were studied by drawing the relationship between them. From these results for selected isotopes, we can see very clearly that the deformation of nuclei decreased when the neutrons number closer to the magic number (N=28) for $^{56}_{28}$Ni, while it was the largest at (N=34) for $^{62}_{28}$Ni isotope.

Keywords: *Deformed nuclei, magic number, electric transition probability ($B(E2)\uparrow$), deformation parameter (β_2), intrinsic electric quadrupole moments (Q_0).*

حساب معلمات التشوه واحتمالية الانتقال الكهربي للنوى زوجية-زوجية لنظائر النيكل $^{54-62}$Ni

مريم عمران مادي[a]، فوزية عمران مادي[b]، و عياد الهادي الزوام[c]
[a, c]قسم الفيزياء، كلية العلوم، جامعة طرابلس، طرابلس، ليبيا
[b]كلية الطيران المدني، مصراتة، ليبيا

الملخص
ركز العمل الحالي على دراسة شكل وخصائص النوى الزوجية-الزوجية التي يكون عددها الكتلي أقل من 100 (A<100) لنظائر ($^{54-62}_{28}$Ni) باستخدام معادلات نموذج القشرة المشوهة. تم الحصول على قيم معامل التشوه (β_2) وعزم رباعي الأقطاب الكهربي الذاتي (Q_0) من حساب احتمالية انتقال رباعي الأقطاب الكهربية المخفضة $B(E2)\uparrow$ من المستوى الأرضي 0^+ إلى الحالة المثارة الأولى 2^+. وبالمقارنة بين القيم المحسوبة لكل من $B(E2)$ و β_2 مع البيانات المعملية التي تم الحصول عليها من المركز الوطني للبيانات النووية (NNDC)، كان التغيير في النتيجة ضئيلاً. كما تمت دراسة العلاقة بين القيم المحسوبة لـ ($B(E2)\uparrow$ و β_2 و Q_0) وعدد النيوترونات عن طريق رسم العلاقة بينهم. ومن خلال هذه النتائج للنظائر المختارة، يمكننا أن نرى بوضوح أن تشوه النوى انخفض عندما اقترب عدد النيوترونات من العدد السحري (N=28) لـ $^{56}_{28}$Ni، بينما كان الأكبر عند (N=34) لنظير $^{62}_{28}$Ni.

الكلمات المفتاحية: *النوى المشوهة، العدد السحري، احتمالية الانتقال الكهربي $B(E2)\uparrow$، معامل التشوه (β_2)، عزم رباعي القطب الكهربي الذاتي (Q_0).*

1. Introduction

In our examination of nuclear radii, we have implicitly assumed that nuclei possess a spherical form. This assumption serves as a reasonable approximation for nuclei that contain magic numbers of neutrons and/or protons: 2, 8, 20, 28, 50, 82, 126. These specific numbers arise from the shell structure of the nucleus, which is similar to the shell structure of atomic electrons. Nuclei characterized by magic numbers of neutrons or protons exhibit a "closed shell" configuration that promotes a spherical shape [1], [2].

Nuclei with proton or neutron numbers far from a magic number are, in general, deformed. The simplest deformations are so-called quadrupole deformations, where the nucleus can take either a prolate shape or an oblate shape, as illustrated in Figure 1 [3].

The warped shell model explains the stability of nuclei that do not conform to magic numbers. The model assumes that nuclei are not always spherical, but may be distorted. Nuclei with a magic number of protons, neutrons, or both are more stable, have higher energy [4], [5].

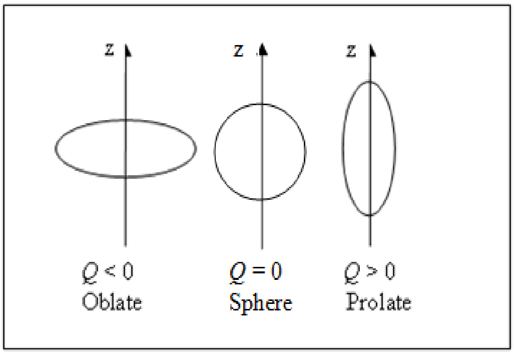

Figure 1. The spheroidal shapes of nuclei as indicated by the value of the electric quadrupole moment

There are many even-even nuclei with neutron or proton numbers close to magic numbers, for example. ^{4}He, ^{8}O, ^{20}Ca, ^{28}Ni, ..., have been studied to investigate their properties and shape. Naz. Jarallah and H. Hassan (2016) [6] studied the shape of $_{54}$Xe and $_{82}Pb$ even-even nuclei, and also calculated the deformation parameters by two different models.

While S. Ebrahiem and H. Zghaier studied deformation parameters of mass-formed nuclei (A=102-178) [7].

A. Ezwam and S. Sallam studied the deformation parameters of $_{58}$Ce even-even isotopes [8].

S. Ibrahim et al calculated the electric transition forces, deformation parameters and mean-squared charge distribution radius of $^{100-124}_{48}$Cd isotopes [9].

In the present work, the aim is to calculate the deformation parameters from the reduced electric quadrupole transition probability for the 0^+ to 2^+ transition for $^{54-62}_{28}$Ni even-even isotopes. Nickel (Ni) has atomic number Z=28 with number of neutrons for these isotopes (that were used in the paper) are N=26, 28, 30, 32, 34, respectively and the mass number $54 \le A \le 62$. We will also study the relationship between the number of neutrons (N) and the deformation parameter (β_2), in addition to knowing the numbers of other neutrons that appear as magic numbers.

2. Theory Part

2.2. The Reduced Electric Quadrupole Transition Probabilities B(E2)

Transformations of radioactive electromagnetic energy between nuclear states represent an ideal approach to understanding nuclear structure and testing models of nuclear structure [10]. The reduced electric quadrupole transition probability between the ground state 0^+ and the first excited 2^+ state in even–even nucleus provide fundamental nuclear information that complements our understanding of the energies of low–lying levels in these nuclei [11], [12].

B(E2)↑ denotes the reduced probability for electric transitions. This reduced probability can be determined using either theoretical or experimental data models for comparison with experimental values [13].

Based on the Global Best Fit systematic, the reduced transition probability B(E2)↑ (in units of (e^2 b^2)) is calculated using the following equation [11]

$$B(E2)\uparrow = \frac{2.6\ Z^2}{E_\gamma\ A^{2/3}}, \quad (1)$$

where:

Z is the atomic number.

E_γ: is the energy of transition from state (I) to the next lower state (I-2), also called the Gamma energies in KeV units. In this case, E_γ of the first excited state (2^+) is required to get a prediction for B(E2)↑.

A: is the mass number of a nucleus.

From Eq. (1), we can show clearly that B(E2)↑ values are basic experimental quantities that do not depend on nuclear models [11].

2.2. Deformation Parameter

Quadrupole deformation parameter can be obtained by the low probability for electric transitions B(E2)↑ by the formula [11]

$$\beta_2 = (4\pi/3ZR_0^2)[B(E2)\uparrow e^2b^2/\ e^2]^{1/2}, \quad (2)$$

where: R_0 is the average nuclear radius. and R_0^2 is in units of barn, it is given by [14]

$$R_0^2 = (1.2\ A^{1/3}\ \ \mathrm{fm})^2 \quad (3)$$

Where:

$1fm = 10^{-15}\ m$, and $1barn = 10^{-28}m^2$.

Then we get

$$R_0^2 = 0.0144\ A^{2/3}\ \ (barn) \quad (4)$$

2.3. Intrinsic Electric Quadrupole Moment

Intrinsic electric quadrupole moment (Q_0) is related to the reduced probability for electric transitions probability B(E2) from 0^+ to 2^+ state by the formula [15]

$$Q_0 = [\ (16\,\pi\ B(E2)\uparrow)/(5\ \ e^2\ )]^{1/2} \quad (5)$$

Where: Q_0 measured in barn (b) unit.
The intrinsic electric quadrupole moment of a spherical nucleus is small and can reach zero in even-even nuclei with magic numbers. Deformed nuclei, located in the middle of the space between closed shells, have large electric moments.

3. Results and Discussions

The experimental values [16] of the first excited state 2^+ (E_γ) in units (KeV), the reduced electric quadrupole transition probabilities B(E2)↑, and deformation parameters β_2 for $^{54-62}_{28}Ni$ even-even isotopes that are given in Table 1.
Also, Table 1 shows the quantities for each isotope that we calculated (present work) for the reduced electric quadrupole transition probabilities B(E2)↑ by using Eq. (1), while deformation parameters β_2 for $^{54-62}Ni$ isotopes were calculated by using Eq. (2), which requires to use of Eq. (4) to calculate the average nuclear radius (R_0^2). Eq. (5) used for calculating the intrinsic electric quadrupole moment (Q_0) for $^{54-62}_{28}Ni$ isotopes.

From Table 1, we noted that the largest value of the reduced probability electric Quadrupole Transition Probabilities B(E2)↑ is (0.1109 e^2b^2) for ($^{62}_{28}Ni$) at (N=34), while the lowest value of B(E2)↑ is for ($^{56}_{28}Ni$) is equal to (0.0516 e^2b^2) at the magic number of neutron (N=28), Figure 2 shows the relation between B(E2)↑ and neutrons number for $^{54-62}_{28}Ni_{26-34}$ even-even isotopes.
By comparing the calculation values with the experimental values for B(E2)↑ for $^{54-62}_{28}Ni$ isotopes in Figure 3, we can note that the calculated values (our work) agree with the experimental values.
Table 1 and Figure 4 show that the number of neutrons (N=26) is close to the midpoint between the two magic numbers (N=20) and (N=28), and for
The reason that a deformation parameter β_2 appears high for ($^{54}_{28}Ni$). Then the deformation value was the lowest (0.1612) at (N=28), and this is a magic number, and then with increasing number of neutrons, the deformation increases until it reaches its highest value is equal to (0.2208) when the number of neutrons is (N=34) for ($^{62}_{28}Ni$), and this number is considered exactly halfway between the numbers (N=28) and (N=40), and for this reason

Table 1 Mass number (A), neutron number (N), the energy transition for first level ($2^+ \rightarrow 0^+$) and the experimental values (B(E2)↑ and β_2) and calculated quantities B(E2)↑, R_0^2 , β_2 and Q_0 respectively for $^{54-62}Ni$ isotopes

A	N	E_γ (KeV)	Experimental		Present Work			
			B(E2) ↑ (e^2b^2)	β_2	B(E2) ↑ (e^2b^2)	R_0^2 (barn)	β_2	Q_0 (barn)
54	26	1392.3	0.061	0.179	0.1025	0.2057	0.2329	1.0151
56	28	2700.6	0.0453	0.151	0.0516	0.2108	0.1612	0.7202
58	30	1454.21	0.0650	0.1768	0.0939	0.2158	0.2121	0.9700
60	32	1332.518	0.0916	0.2052	0.0998	0.2207	0.2141	1.0017
62	34	1172.91	0.0889	0.1969	0.1109	0.2256	0.2208	1.0559

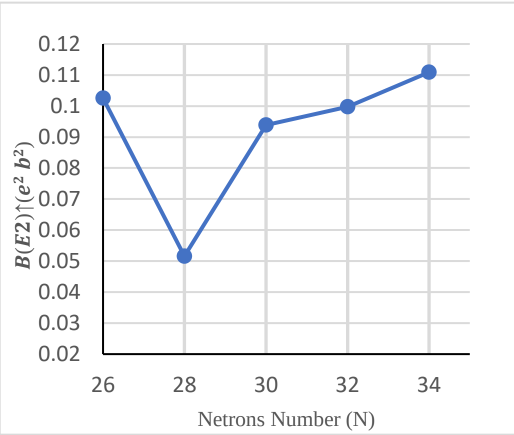

Figure 2 The variation between the reduced probability electric quadrupole transition Probabilities B(E2)↑ and neutrons number (N) for $^{54-62}_{28}Ni$ isotopes

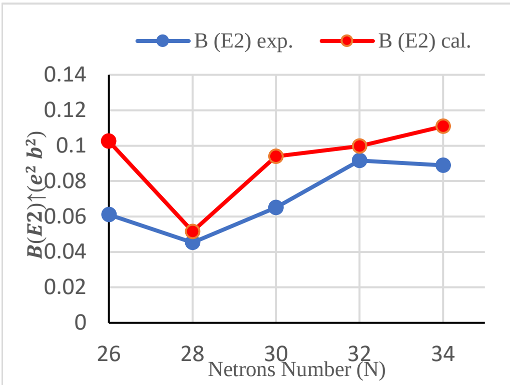

Figure 3 Comparison between B(E2)↑$_{exp.}$ and B(E2)↑$_{p.\ w.}$ values as a function of neutrons number (N) for $^{54-62}_{28}Ni$ isotopes

(N=40) is considered a magic number for the nickel

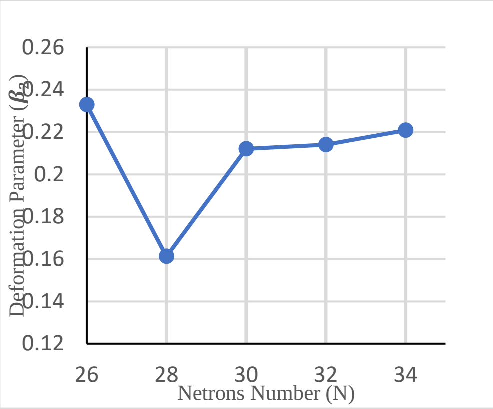

Figure 4 The variation between deformation parameters β_2 and neutrons number (N) for $^{54-62}_{28}Ni$ isotopes

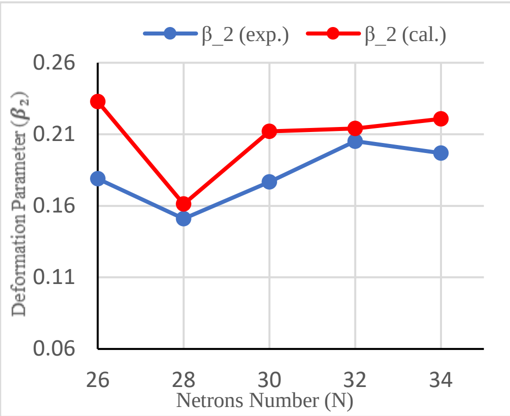

Figure 5 Comparison between β_2 for exp. and β_2 for p. w. values as a function of neutrons number (N) for $^{54-62}_{28}Ni$ isotopes

nucleus.

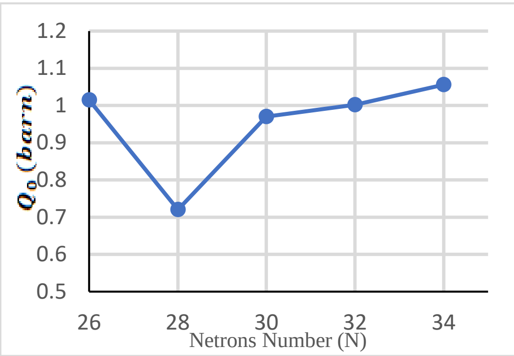

Figure 6 The relation between intrinsic electric quadrupole moment Q_0 and neutrons number (N) for $^{54-62}_{28}Ni$ isotopes

From Table 1 and Figure 5, we can notice less agreement between the calculated values and the experimental values for deformation parameter β_2 value for $^{54}_{28}Ni$ at (N=26), while better agreement for others. Also, by using the calculation results (in Table 1) for the intrinsic electric quadrupole moment Q_0, we observe that the lowest value of Q_0 is equal (0.7202 barn) for $^{56}_{28}Ni$ at a magic number (N=28), and as large as (N=34) for $^{62}_{28}Ni$ equal to Q_0 = 1.0559 barn that gives in Figure 6.

4. Conclusion

From the current work and our results that were obtained for the values of the reduced probability electric quadrupole transition probabilities B(E2)↑, deformation parameter β_2, and the intrinsic electric quadrupole moment Q_0, it can be concluded that nuclei with some neutrons and/or protons close to the magic numbers are more stable, less deformation, and nearly spherical in shape. In other words, the values of deformation (β_2) become as low as possible, and, therefore, this isotope with magic numbers is more stable than others. Therefore, the nickel nucleus is considered one of the spherical nuclei, the least deformed, the most loosely bound, and the most stable, especially at a magic number. While the other nuclei isotopes that have the neutron number in the midpoint between two magic numbers, these nuclei isotopes will be less stable, non-spherical shape and will have large deformation, also be more elongated.

Finally, in the next study, other even-even Ni isotopes and other nuclei can be studied to know the shape and properties of nuclei.

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