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Existence of a Local Solution for an Attraction-Repulsion Chemotaxis System with Nonlinear Diffusion and a Logistic Source

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A B S T R A C T
In this paper, we are concerned with a model arising from biology, which is a coupled system of chemotaxis equations and viscous compressible fluid equations through transport and external forcing. The local existence of solutions to the Cauchy problem is investigated under certain conditions. Precisely, for an attraction-repulsion chemotaxis model system over three space dimensions, we obtain local existence and uniqueness of convergence on classical solutions near constant states. We prove local existence of unique solutions in three dimensions by using energy estimates.

Keywords: *chemotaxis, local existence, logistic source, nonlinear diffusion.*

وجود حل محلي لنظام الجذب الكيميائي والتنافر مع الانتشار غير الخطي والمصدر اللوجستي

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الملخص
في هذا البحث، نحن معنيون بنموذج ناشئ من علم الأحياء، وهو نظام مقترن من معادلات الانجذاب الكيميائي ومعادلات الموائع اللزوجة القابلة للانضغاط من خلال النقل والتأثير الخارجي. يتم التحقيق في الوجود المحلي لحلول مشكلة كوشي في ظل ظروف معينة. على وجه التحديد، من خلال نظام نموذجي للتجاذب والتنافر الكيميائي على ثلاثة أبعاد فضائية، حصلنا على الوجود المحلي وتفرد التقارب في الحلول الكلاسيكية بالقرب من الحالات الثابتة. نثبت الوجود المحلي لحلول فريدة في ثلاثة أبعاد باستخدام تقدير الطاقة.

الكلمات المفتاحية: الانجذاب الكيميائي، الوجود المحلي، المصد اللوجستي، الانتشار غير الخطي.

1. Introduction

Chemotaxis is the process of directed movement of organisms toward a higher or lower concentration of a particular chemical. It plays a crucial role in many biological phenomena, such as immune system response, embryonic development, tumor growth, population dynamics, and gravitational collapse. In 1970, Keller and Segel proposed a famous mathematical model to describe the phenomenon of chemotaxis [1,2]. Since these interesting works of Keller and Segel, many variants of chemotaxis models have been proposed and their dynamics have been intensively studied, mainly in homogeneous environments [3,4, 5, 6, 7,8, 9] and [10,11,12, 13, 14, 15]. It is based on the H^N a priori estimate with time-weighted functions by the energy method [16,17, 18, 19]. In particular, we refer to survey papers [20,21]. In fact, the underlying environments of many biological systems undergo various spatial and temporal changes. Studying chemotaxis models in heterogeneous environments has biological and mathematical interest. Huang et al. [22] obtained the initial-boundary value problem of a chemotaxis system with singular sensitivity and logistic. In addition, they studied the local existence of solutions under different conditions. Xiao and Li [23] established the existence of a weak solutions to the static problem, via the potential. Matsumura and Nishida [24] obtained the initial-value problem for equations of motion of viscous and heat conductive gases in three dimensions, and they obtained the existence of global solutions in H^3 with a method based on iteration and the energy method. However, important dynamical questions, including the uniqueness and global stability of fully positive solutions, remain open for models of chemotaxis systems. In this paper, we study a mathematical model for the motion of swimming bacteria, eukaryotes in compressible viscous fluid in $\mathbb{R}^3$given by

$$\begin{aligned}
&\partial_t n + \nabla \cdot (nu) = \delta \Delta n + n(n_\infty - n) \\
&\partial_t u + u.\nabla u + \frac{\nabla p(n)}{n} = \nabla h_1 - \nabla h_2 + \lambda u \\
&\partial_t h_1 = \Delta h_1 - k_1 h_1 + k_2 n \\
&\partial_t h_2 = \Delta h_2 - k_3 h_2 + k_4 n, \qquad (1.1)
\end{aligned}$$

where $n(x,t), u(x,t), h_1(x,t), h_2(x,t), p(n)$, for $t > 0, x \in \mathbb{R}^3$, are the density of cells, velocity of cells, chemoattractant concentration, chemorepellent concentration, and the pressure of the fluid . A positive constant λ is the coefficient of the damping. The initial data is given by

$(n, u, h_1, h_2)|_{t=0} = (n_0, \ u_0, h_{1,0}, h_{2,0})(x), \ \ x \in \mathbb{R}^3$,

Where it is supposed to hold that $(n_0, \ u_0, h_{1,0}, h_{2,0})(x) \longrightarrow (\mathrm{n}_\infty, 0 \ , h_{1,\infty}, h_{2,\infty})$, $as\ |x| \to \infty$.

The aim of this paper is to study the local existence and uniqueness of the positive solutions of the chemotaxis model with logistic sources. The main goal of this paper is to establish the local existence of smooth solutions in three dimensions around a constant state $(\mathrm{n}_\infty, 0 \ , h_{1,\infty}, h_{2,\infty})$.

The main result of this paper is stated as follows:

Theorem 1.1. Let $N \geq 4$ be an integer and $\mathrm{U(t)} = [n, u, h_1, h_2]$ be a smooth solution to the Cauchy problem of the chemotaxis system (1.1) with initial data $\mathrm{U}_0 = [n_0, u_0, h_{1,0}, h_{2,0}]$. Suppose that there is a sufficiently small constant δ_0 such that

$$\|\mathrm{U}_0\|_{H^N} \leq \delta_0,$$

then the Cauchy problem (1.1) - (1.2) admits a unique classical solution U(t) locally in time which satisfies

$$\begin{aligned}
&(n - n_\infty, h_1 - h_{1,\infty}, h_2 - h_{2,\infty})(t) \in \\
&C([0,\infty); H^N(\mathbb{R}^3)) \cap C^1([0,\infty); H^{N-2}(\mathbb{R}^3)), \\
&\quad u \in \ C([0,\infty); H^N(\boldsymbol{R}^3)) \cap C^1([0,\infty); H^{N-1}(\mathbb{R}^3)). \qquad (1.3)
\end{aligned}$$

We present some assumptions which will be used in the rest of the paper; that is

$$n_0 \ \geq 0, \ u_0 \geq 0, \ h_{1,0} \geq 0, h_{2,0} \geq 0.$$

For an integrable function $f: R^3 \to R$, let us denote the space

$$X(0,T) = \left\{ \begin{array}{l} (n - n_\infty, h_1 - h_{1,\infty}, h_2 - h_{2,\infty}) \in \\ C([0,T]; H^N(\boldsymbol{R}^3)) \cap C^1([0,T]; H^{N-2}(\mathbb{R}^3), \\ u \in C\left([0,T]; H^N(\mathbb{R}^3) \cap C^1(0,T); H^{N-1}(\mathbb{R}^3)\right) \end{array} \right\}.$$

This paper is organized as follows. In section 2, we introduce some notation and definitions in this paper. In section 3, we prove the local existence and uniqueness of the solutions.

2. Notation and preliminaries.

Throughout this paper, we introduce some notation for later use. Let Ω be a bounded domain in $\mathbb{R}^3$. c denotes a general constant, c_i where $i = 1,2$, denotes some positive (generally small) constant, which may take different values in different places. We also set $\partial^\alpha = (\partial_{x_1}^{\alpha_1}, \partial_{x_2}^{\alpha_2}, \partial_{x_3}^{\alpha_3})$ for a multi-index $\alpha = (\alpha_1, \alpha_2, \alpha_3)$.

Definition 2.1

If $u = (P, Q, R)$ is a vector field in $\mathbb{R}^3$and P, Q, and R all exist, then the divergence of u is defined by

$\text{div u} = \mathrm{P_x} + Q_y + R_z = \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$.

For simplicity, the divergence of u can be written as the dot product

$\text{div u} = \nabla.\mathrm{u}$.

Definition 2.2

A function of class $C^k(\Omega)$ is a function of smoothness at least k; that is, a function of class $C^k(\Omega)$ is a function that has a kth derivative that is continuous in Ω:

$$C^k(\Omega) = \{f: \Omega \to \mathbb{R} \mid f \text{ is k} - \text{times continuously differentiable}\}$$

A function of class $C^\infty(\Omega)$ or $C^\infty(\Omega)$-function is an infinitely differentiable function; that is, a function that has derivatives of all orders in Ω:

$$C^\infty(\Omega) = \{f: \Omega \to \mathbb{R} \mid \ f \text{ is infinitely differentiable} \ \} \cdot$$

Definition 2.3

For $1 \leq p < \infty$, the space $L^p(\Omega)$ consists of the Lebesgue measurable functions $f: \Omega \to \mathbb{R}$ such that

$$\int_{\Omega} |f(x)|^p \, dx < \infty,$$

and $L^{\infty}(\Omega)$ consists of the essentially bounded function.

These spaces are Banach spaces with respect to the norms

$$\|f\|_{L^P} = \left(\int_{\Omega} |f(x)|^p \, dx\right)^{\frac{1}{p}}, \|f\|_{\infty} = \sup_{x\in\Omega} |f|$$

where sup denotes the essential supremum,

$$\sup_{x\in\Omega} |f| = \inf \{M \geq 0 : \ |f| \leq M \ \text{almost evrywhere in } \Omega\}.$$

Definition 2.4

The Sobolev space $W^{N,p}(U)$ consists of all summable functions $u: U \to \mathbb{R}$ such that for each multi-index α with $|\alpha| \leq k$, $D^{\alpha}u$ exists in the weak sense and belongs to $L^p(U)$.

Definition 2.5

If $u \in W^{N,p}(U)$, we define its norm to be

$$\|u\|_{W^{N,p}(U)} = \begin{cases} \left(\sum_{|\alpha|\leq k} \int_{\mathrm{U}} |D^{\alpha}u|^p \, dx\right)^{\frac{1}{p}} & 1 \leq p < \infty \\ \sum_{|\alpha|\leq k} ess\ sup|D^{\alpha}u| & p = \infty. \end{cases}$$

The Sobolev space $W^{N,2}(\mathbb{R}^3)$ is denoted as H^N. When $N = 0$, the norms in the space $L^2(\mathbb{R}^d)$ are denoted by $\|\cdot\|$. We will denote $\|\cdot\|_N$ is the H^N norm.

Cauchy`s inequality with ϵ

Let a, b are any real numbers. Then

$$ab \leq \epsilon a^2 + \frac{b^2}{4\varepsilon} \qquad a > 0, b > 0, \epsilon > 0.$$

Cauchy-schwarz inequality

$|x + y| \leq |x||y| \qquad (x, y \in \mathbb{R}^n).$

Gronwall's inequality (differential form)

Let $f(t)$ be a nonnegative, absolutely continuous function $[0, T]$, which satisfies the differential inequality

$$f'(t) \leq g(t)f(t) + k(t),$$

where $g(t)$ and $k(t)$, are nonnegative, summable functions on $[0, T]$. Then,

$$f(t) \leq e^{\int_0^t g(s)ds}[f(0) + \int_0^t k(s)ds]$$

for all $0 \leq t \leq T$.

3. Existence of local solutions

Let U(t) = $[n, u, h_1, h_2]$ be a smooth solution to the Cauchy problem of the chemotaxis system (1.1) with initial data $U_0 = [n_0, u_0, h_{1,0}, h_{2,0}]$. To prove the local existence of solutions to the Cauchy problem (1.1) when initial data is a small, smooth perturbation near the steady-state $(\mathrm{n}_{\infty}, 0, h_{1,\infty}, h_{2,\infty})$, let us take changes of variables

$$n(x,t) = \sigma(x,t) + n_{\infty}, \ h_1 = v + h_{1,\infty}$$

$$and \ \ h_2 = w + h_{2,\infty}.$$

So, the Cauchy problem (1.1) is reformulated as

$$\begin{aligned} &\partial_t\sigma + n_{\infty}\nabla\cdot u - \delta\Delta\sigma + n_{\infty}\sigma = -u.\nabla\sigma - \sigma.\nabla u - \sigma^2 \\ &\partial_t u - \lambda u = -u.\nabla u - \frac{p'(\sigma + n_{\infty})}{\sigma + n_{\infty}}\nabla\sigma + \nabla v - \nabla w \\ &\partial_t v = \Delta v - k_1 v + k_2\sigma \qquad (3.1) \\ &\partial_t w = \Delta w - k_3 w + k_4\sigma, \end{aligned}$$

with initial data

$$(\sigma, u, v, w)|_{t=0} = (\sigma_0, u_0, v_0, w_0) \to (0,0,0,0) \text{ as } |x| \to \infty \qquad (3.2)$$

Where $\sigma_0 = n_0 - n_{\infty}$, $v_0 = h_{1,0} - h_{1,\infty}$ and $w_0 = h_{2,0} - h_{2,\infty}$, which satisfies the compatibility condition $k_2 n_{\infty} - k_1 h_{1,\infty} = 0$ and $k_4 n_{\infty} - k_3 h_{2,\infty} = 0$.

Now, we construct a solution sequence $(n^j, u^j, {h_1}^j, {h_2}^j)_{j\geq 0}$ by solving iteratively the Cauchy problem for the following

$$\begin{aligned} &\partial_t\sigma^{j+1} + n_{\infty}\nabla\cdot u^{j+1} - \delta\Delta\sigma^{j+1} + n_{\infty}\sigma^{j+1} = -\nabla(\sigma^{j+1}.u^j) - {\sigma^j}^2 \\ &\partial_t u^{j+1} - \lambda u^{j+1} = -u^j.\nabla u^j - \frac{p'(\sigma^j + n_{\infty})}{\sigma^j + n_{\infty}}\nabla\sigma^{j+1} + \nabla v^{j+1} - \nabla w^{j+1} \\ &\partial_t v^{j+1} = \Delta v^{j+1} - k_1 v^{j+1} + k_2\sigma^j \qquad (3.3) \\ &\partial_t w^{j+1} = \Delta w^{j+1} - k_3 w^{j+1} + k_4\sigma^j \end{aligned}$$

with

$$(\sigma^{j+1}, u^{j+1}, v^{j+1}, w^{j+1})\big|_{t=0} = (\sigma_0, u_0, v_0, w_0) \to (0,0,0,0) \text{ as } |x| \to \infty, \qquad (3.4)$$

for j > 0. In what follows, let us write $A^j = (\sigma^j, u^j, v^j, w^j)$ and $A_0 = (\sigma_0, u_0, v_0, w_0)$, where A^0 = (0, 0, 0, 0) for simplicity.

Now, we can start the following Lemma:

Lemma 3.1 There are constants $T_1 > 0$, $\varepsilon_0 > 0$, $B > 0$ such that if the initial data $A_0 \in H^N(\mathbb{R}^3)$ and $\|A_0\|_N \leq \varepsilon_0$, then for each $j \geq 0$, $A^j \in$ C([0, T_1]; $H^N(\mathbb{R}^3)$) is well defined and

$$\sup_{0\leq t\leq T_1} \left\|A^j(t)\right\|_N \leq B. \qquad (3.5)$$

Moreover, $(A^j)_{j\geq 0}$ is a Cauchy sequence in the Banach space C ([0, T_1]; $H^N(\mathbb{R}^3)$), which converges to the solution A = (σ, u, v, w) of the Cauchy problem (3.1) - (3.2), and satisfies

$$\sup_{0\leq t\leq T_1} \|A(t)\|_N \leq B. \quad (3.6)$$

Finally, the Cauchy problem (3.1) -(3.2) admits at most one solution in C ([0, T_1]; H^N ($\mathbb{R}^3$)), which satisfies (3.6).

Proof. We take A^0 = (0,0,0,0). Then, we use that to solve the equations for A^1. The first, third, and fourth equations are the second order parabolic equations. The second equation is the wave equation. We obtain $\sigma^1(x,t), v^1(x,t), w^1(x,t)$, and $u^1(x,t)$ in this order. Similarly, we define $(\sigma^j, v^j, w^j, u^j)$ iteratively. Now, we prove the existence and uniqueness of solutions in the space $C([0,T_1]; H^N(\mathbb{R}^3))$, where $T_1 > 0$, is suitably small. The proof is divided into four steps as follows.

First step. We show the uniform boundedness of the sequence of functions under our construction by energy estimates. We use induction to prove (3.5). We observe the case j = 0 by the assumption A^0 = (0,0,0,0). Suppose that it is true for $j \geq 0$ with B small enough. To prove for $j+1$, we need some energy estimates for A^{j+1}.

Applying ∂^α to the first equation of (3.3), multiplying it by $\partial^\alpha \sigma^{j+1}$, and integrating in x, we get

$$\int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha \sigma_t^{j+1} dx + n_\infty \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha \sigma^{j+1} dx - \delta \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1}\, \partial^\alpha \Delta \sigma^{j+1} dx = -n_\infty \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha \nabla . u^{j+1} dx - \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1}\, \partial^\alpha \nabla(\sigma^{j+1}. u^j) dx - \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1}\, \partial^\alpha {\sigma^j}^2 dx.$$

It follows that

$$\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3} \left|\partial^\alpha \sigma^{j+1}\right|^2 dx + n_\infty \int_{\mathbb{R}^3} \left|\partial^\alpha \sigma^{j+1}\right|^2 dx - \delta \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha \Delta \sigma^{j+1} dx = - n_\infty \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha\, \nabla \cdot u^{j+1}\, dx - \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha\, \nabla(\sigma^{j+1}. u^j) dx - \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha\, \sigma^{j2} dx.$$

By using integration by parts, one has

$$\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3} \left|\partial^\alpha \sigma^{j+1}\right|^2 dx + n_\infty \int_{\mathbb{R}^3} \left|\partial^\alpha \sigma^{j+1}\right|^2 dx + \delta \int_{\mathbb{R}^3} \partial^\alpha \nabla \sigma^{j+1}\, \partial^\alpha \nabla \sigma^{j+1}\, dx = -n_\infty \int_{\mathbb{R}^3} \partial^\alpha \nabla \sigma^{j+1} \partial^\alpha\, u^{j+1} dx + \int_{\mathbb{R}^3} \partial^\alpha \nabla \sigma^{j+1} \partial^\alpha\, (u^j \cdot \sigma^{j+1}) dx + \int_{\mathbb{R}^3} \partial^\alpha \sigma^{j+1} \partial^\alpha\, \sigma^{j2} dx\, .$$

The right-hand side of the previous equation is bounded by

$$\frac{n_\infty}{2}\left\|u^{j+1}\right\|_N \left\|\nabla \sigma^{j+1}\right\|_N + c\left\|u^j\right\|_N \left\|\sigma^{j+1}\right\|_N \left\|\nabla \sigma^{j+1}\right\|_N + c\left\|\sigma^j\right\|_{N-2} \left\|\sigma^{j+1}\right\|_N \left\|\sigma^j\right\|_N.$$

Then, we obtain

$$\frac{1}{2}\frac{d}{dt}\left\|\sigma^{j+1}\right\|_N^2 + n_\infty \left\|\sigma^{j+1}\right\|_N^2 + \delta\left\|\nabla \sigma^{j+1}\right\|_N^2 \leq \frac{n_\infty}{2}\left\|\nabla \sigma^{j+1}\right\|_N^2 + \frac{n_\infty}{2}\left\|u^{j+1}\right\|_{H^N}^2 + c\left\|u^j\right\|_{H^N}^2 \left\|\sigma^{j+1}\right\|_N^2 + c\left\|\nabla \sigma^{j+1}\right\|_N^2 + c\left\|\sigma^j\right\|_N^2 \left\|\sigma^{j+1}\right\|_N^2 + \left\|\sigma^j\right\|_N^2 . \quad (3.7)$$

Applying ∂^α to the second equation of (3.3), multiplying it by $\partial^\alpha u^{j+1}$and then integrating in x, we have

$$\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^3} (\partial^\alpha u^{j+1})^2 dx - \lambda \int_{\mathbb{R}^3} (\partial^\alpha u^{j+1})^2 dx + \frac{p'(n_\infty)}{n_\infty}\int_{\mathbb{R}^3} \partial^\alpha\, u^{j+1}\, \partial^\alpha \nabla \sigma^{j+1} dx = -\int_{\mathbb{R}^3} \partial^\alpha u^{j+1} \partial^\alpha\, (u^j \nabla \cdot u^j)\, dx - \int_{\mathbb{R}^3} \partial^\alpha u^{j+1} \partial^\alpha \left(\frac{p'(\sigma^j + n_\infty)}{\sigma^j + n_\infty} - \frac{p'(n_\infty)}{n_\infty}\right) \nabla \sigma^{j+1} dx + \int_{\mathbb{R}^3} \partial^\alpha u^{j+1} \partial^\alpha\, \nabla v^{j+1}\, dx - \int_{\mathbb{R}^3} \partial^\alpha u^{j+1} \partial^\alpha\, \nabla w^{j+1} dx.$$

Then, after using integration by parts, taking the summation over $|\alpha| \leq N$, and using the Cauchy inequality, we get

$$\frac{1}{2}\frac{d}{dt}\left\|u^{j+1}\right\|_N^2 + \lambda\left\|u^{j+1}\right\|_N^2 \leq \frac{p'(n_\infty)}{2n_\infty}\left\|\nabla \sigma^{j+1}\right\|_N^2 + \frac{p'(n_\infty)}{2n_\infty}\left\|u^{j+1}\right\|_N^2 + c\left\|u^j\right\|_N^2 \left\|u^{j+1}\right\|_N^2 + c\left\|u^j\right\|_N^2 + c\left\|\sigma^j\right\|_N^2 \left\|u^{j+1}\right\|_N^2 + c\left\|\nabla \sigma^{j+1}\right\|_N^2 + c\left\|u^{j+1}\right\|_N^2 + c\left\|\nabla v^{j+1}\right\|_N^2 + c\left\|\nabla w^{j+1}\right\|_N^2. \quad (3.8)$$

Similarly, for the estimate v and w, we have

$$\frac{1}{2}\frac{d}{dt}\left\|v^{j+1}\right\|_N^2 + c\left\|v^{j+1}\right\|_N^2 + \left\|\nabla v^{j+1}\right\|_N^2 \leq c\left\|\sigma^j\right\|_N^2 \left\|v^{j+1}\right\|_N^2 \quad (3.9)$$

$$\frac{1}{2}\frac{d}{dt}\left\|w^{j+1}\right\|_N^2 + c\left\|w^{j+1}\right\|_N^2 + \left\|\nabla w^{j+1}\right\|_N^2 \leq c\left\|\sigma^j\right\|_N^2 \left\|w^{j+1}\right\|_N^2. \quad (3.10)$$

We combine the equations (3.7) -(3.10) to get

$$\frac{1}{2}\frac{d}{dt}\left(\|\sigma^{j+1}\|_N^2+\|u^{j+1}\|_N^2+\|v^{j+1}\|_N^2+\|w^{j+1}\|_N^2\right) + c_1\left(\|\sigma^{j+1}\|_N^2+\|u^{j+1}\|_N^2 + \|v^{j+1}\|_N^2+\|w^{j+1}\|_N^2\right) + c_2(\|\nabla\sigma^{j+1}\|_N^2+\|\nabla v^{j+1}\|_N^2 + \|\nabla w^{j+1}\|_N^2) \le c(\|\sigma^j\|_N^2 + \|u^j\|_N^2) + c\|\sigma^j,u^j\|_N^2(\|\sigma^{j+1}\|_N^2 + \|u^{j+1}\|_N^2+\|v^{j+1}\|_N^2 + \|w^{j+1}\|_N^2.$$

Thus, after integrating with respect to t, we get

$$\|A^{j+1}(t)\|_N^2 + c_1\int_0^t\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2ds + c_2\int_0^t\nabla\cdot\|\sigma^{j+1},v^{j+1},w^{j+1}\|_N^2ds \le c\|A^{j+1}(0)\|_N^2 + c\int_0^t\|A^j(s)\|_N^2ds + \int_0^t\|A^j(s)\|_N^2\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2, \quad (3.11)$$

which from the inductive assumption implies

$$\|A^{j+1}(t)\|_N^2 + c_1\int_0^t\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2ds + c_2\int_0^t\nabla\cdot\|\sigma^{j+1},v^{j+1},w^{j+1}\|_N^2ds \le c\epsilon_0^2+cB^2T_1+cB^2\int_0^t\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2.$$

Next, taking the small constants $\epsilon_0>0$, $T_1>0$ and M > 0 for $0\le t\le T_1$. Then we have

$$\|A^{j+1}(t)\|_N^2 + c_1\int_0^t\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2ds + c_2\int_0^t\nabla\|\sigma^{j+1},v^{j+1},w^{j+1}\|_N^2ds \le B^2. \quad (3.12)$$

From this, we deduce that (3.5) holds for j + 1 and $0\le t\le T_1$. Hence, (3.5) is proved for all j≥ 0.

For the second step, we show that $\|A^{j+1}(t)\|_N^2$ is continuous in time for each j ≥ 0. For simplicity, let us define the equivalent energy function

$$E\left(A^{j+1}(t)\right)=\|\sigma^{j+1}\|_N^2+\|u^{j+1}\|_N^2+\|v^{j+1}\|_N^2 + \|w^{j+1}\|_N^2.$$

Similarly, to how we proved (3.12), we have

$$|EA^{j+1}(t)-EA^{j+1}(s)| = \left|\int_s^t\frac{d}{d\theta}E(A^{j+1}(\theta))\,d\theta\right| \le c\int_s^t\|A^j(\theta)\|_N^2d\theta + c\int_s^t(1+\|A^j(\theta)\|_N^2)\|\sigma^{j+1},u^{j+1},w^{j+1},v^{j+1}\|_N^2d\theta + c\int_s^t\nabla\cdot\|\sigma^{j+1},v^{j+1},w^{j+1}\|_N^2d\theta \le cB^2(t-s) + c(B^2+1)\int_s^t\|\sigma^{j+1},u^{j+1},v^{j+1},w^{j+1}\|_N^2d\theta + c\int_s^t\nabla\cdot\|\sigma^{j+1},v^{j+1},w^{j+1}\|_N^2d\theta, \quad (3.13)$$

for any $0\le s\le t\le T_1$. The time integral on the right-hand side from the above inequality is bounded by (3.12). Then, we conclude that $E(A^{j+1}(t))$ is continuous in t for each j ≥ 0. Thus, we claim that $\|A^j(t)\|_N^2$ is continuous in time for each j ≥ 1.

Third step, we prove that the sequence $(A^j)_{j\ge0}$ is a Cauchy sequence in the Banach space C ([0, T_1]; $H^N(\mathbb{R}^3)$), which converges to the solution A = (σ,u,v,w) of the Cauchy problem (3.2) - (3.3), and satisfies

$$\sup_{0\le t\le T_1}\|A(t)\|_N\le B.$$

Subtract the j-th equations from the (j + 1)-th equations. For simplicity, we denote $\delta g^j=g^{j+1}-g^j$. We have the following equations for $\delta\sigma^j$, δu^j, δv^j and δw^j :

$$\begin{aligned}
&\partial_t\delta\sigma^{j+1}+n_\infty\delta\sigma^{j+1}-\Delta\delta\sigma^{j+1} \\
&=-n_\infty\nabla\cdot\delta u^{j+1}-u^j\nabla\cdot\delta\sigma^{j+1}-\delta u^j\nabla\cdot\sigma^j - \\
&-\sigma^j\nabla\cdot\delta u^j-\delta\sigma^j\nabla\cdot u^j+(\sigma^j+\sigma^{j-1})\delta\sigma^j \\
&\partial_t\delta u^{j+1}+\lambda\delta u^{j+1}=-u^j\nabla\cdot\delta u^j-\delta u^j\,\nabla\cdot u^{j-1} \\
&+\nabla\delta v^{j+1}\ -\nabla\delta w^{j+1} \\
&+\frac{\nabla p(\sigma^j+n_\infty)}{\sigma^j+n_\infty}-\frac{\nabla p(\sigma^{j-1}+n_\infty)}{\sigma^{j-1}+n_\infty} \\
&\partial_t\delta v^{j+1}=\Delta\delta v^{j+1}-k_1\delta v^{j1+}+k_2\,\delta\sigma^j \\
&\partial_t\delta w^{j+1}=\Delta\delta w^{j+1}-k_3\delta w^{j+1}+k_4\,\delta\sigma^j.
\end{aligned}$$

(3.14)

The estimate for $\delta\sigma^{j+1}$ is

$$\frac{1}{2}\frac{d}{dt}\|\delta\sigma^{j+1}\|_N^2+n_\infty\|\delta\sigma^{j+1}\|_N^2+c\|\nabla\delta\sigma^{j+1}\|_N^2 \le c\|\delta u^{j+1}\|_N^2+c\|\delta\sigma^{j+1}\|_N^2 + c\|u^j\|_N\|\delta\sigma^{j+1}\|_N\|\nabla\delta\sigma^{j+1}\|_N + c\|\sigma^j\|_N\|\delta\sigma^{j+1}\|_N\|\delta u^j\|_N + c\|\sigma^j\|_N\|\nabla\delta\sigma^{j+1}\|_N\|\delta u^j\|_N + c\|u^j\|_N\|\delta\sigma^j\|_N\|\delta\sigma^{j+1}\|_N + c\|\delta\sigma^j\|_N\|\delta\sigma^{j+1}\|_N.$$

It follows that

$$\frac{1}{2}\frac{d}{dt}\|\delta\sigma^{j+1}\|_N^2+c\|\delta\sigma^{j+1}\|_N^2+c\|\nabla\delta\sigma^{j+1}\|_N^2 \le c\|\delta u^{j+1}\|_N^2+c\|\delta\sigma^{j+1}\|_N^2+c\|\delta u^j\|_N^2$$

$$+ c\|u^j\|_N^2\|\delta\sigma^{j+1}\|_N^2 + c\|\sigma^j\|_N^2\|\delta\sigma^{j+1}\|_N^2$$
$$+c\|\sigma^j\|_N^2\|\nabla\delta\sigma^{j+1}\|_N^2 + c\|u^j\|_N^2\|\delta\sigma^{j+1}\|_N^2$$
$$+ c\|\delta\sigma^j\|_N^2 + c\|\delta\sigma^{j+1}\|_N^2. \quad (3.15)$$

The estimate for δu^{j+1} is

$$\frac{1}{2}\frac{d}{dt}\|\delta u^{j+1}\|_N^2 + \lambda\|\delta u^{j+1}\|_N^2$$
$$\leq c\|u^j\|_N\|\delta u^{j+1}\|_N\|\delta u^j\|_N^2$$
$$+ c\|\delta u^{j+1}\|_N\|\nabla\delta v^{j+1}\|_N$$
$$+ c\|\delta u^{j+1}\|_N\|\nabla\delta w^{j+1}\|_N$$
$$+ c\|\delta u^{j+1}\|_N^2\|\nabla\delta\sigma^j\|_N^2.$$

Then, we have

$$\frac{1}{2}\frac{d}{dt}\|\delta u^{j+1}\|_N^2+\lambda\|\delta u^{j+1}\|_N^2$$
$$\leq c\|u^j\|_N^2\|\delta u^{j+1}\|_N^2 + \|\delta u^j\|_N^2 +$$
$$+c\|\nabla\delta v^{j+1}\|_N^2 + c\|\nabla\delta w^{j+1}\|_N^2 +$$
$$c\|\nabla\delta\sigma^j\|_N^2. \quad (3.16)$$

In a similar way as above, we estimate δv^{j+1} and δw^{j+1} as follows:

$$\frac{1}{2}\frac{d}{dt}\|\delta v^{j+1}\|_N^2 + \|\nabla\delta v^{j+1}\|_N^2 + k_1\|\delta v^{j+1}\|_N^2 \leq$$
$$c\|\delta\sigma^j\|_N^2 + c\|\delta v^{j+1}\|_N^2 \quad (3.17)$$

and

$$\frac{1}{2}\frac{d}{dt}\|\delta w^{j+1}\|_N^2 + \|\nabla\delta w^{j+1}\|_N^2 + k_3\|\delta w^{j+1}\|_N^2 \leq$$
$$c\|\delta\sigma^j\|_N^2 + c\|\delta w^{j+1}\|_N^2. \quad (3.18)$$

Combine equations (4.15) - (4.18) to get

$$\frac{1}{2}\frac{d}{dt}(\|\delta\sigma^{j+1}\|_N^2 + \|\delta u^{j+1}\|_N^2 + \|\delta v^{j+1}\|_N^2$$
$$+\|\delta w^{j+1}\|_N^2) + c_1\left(\|\delta\sigma^{j+1}\|_N^2 + \|\delta u^{j+1}\|_N^2 +\right.$$
$$\left.\|\delta v^{j+1}\|_N^2 + \|\delta w^{j+1}\|_N^2\right) + c_2\left(\|\nabla\delta\sigma^{j+1}\|_N^2 +\right.$$
$$\left.\|\nabla\delta v^{j+1}\|_N^2 + \|\nabla\delta w^{j+1}\|_N^2\right) \leq c\left(\|\delta\sigma^{j+1}\|_N^2 +\right.$$
$$\left.\|\delta u^{j+1}\|_N^2 + \|\delta v^{j+1}\|_N^2 + \|\delta w^{j+1}\|_N^2\right) +$$
$$c\left(\|\delta\sigma^j\|_N^2 + \|\delta u^j\|_N^2\right).$$

Using Gronwall's inequality, we obtain

$$\sup_{0\leq t\leq T_1}(\|\delta\sigma^{j+1}\|_N^2 + \|\delta u^{j+1}\|_N^2 + \|\delta v^{j+1}\|_N^2 + \|\delta w^{j+1}\|_N^2)$$
$$\leq e^{\int_0^t cds}\int_0^t\|\delta A^j(s)\|_N^2 ds$$
$$+ e^{\int_0^t cds}\int_0^t\|\delta A^{j+1}(0)\|_N^2 ds$$
$$\leq cT_1 e^{cT_1}\sup_{0\leq t\leq T_1}\|\delta A^j\|_N^2. \quad (3.19)$$

We take $T_1 > 0$ sufficiently small, then we find that $(A^j)_{j\geq 0}$ is a Cauchy sequence in the Banach space C $([0, T_1]; H^N(\mathbb{R}^3))$. Thus, the limit function

$$A = A^0 + \lim_{m\to\infty}\sum_{j=1}^{m}(A^{j+1} - A^j),$$

in the same space C $([0, T_1]; H^N(\mathbb{R}^3))$ and satisfies

$$\sup_{0\leq t\leq T_1}\|A\|_N \leq \sup_{0\leq t\leq T_1}\lim_{j\to\infty}\inf\|A^j\|_N \leq B.$$

For the fourth step, we show that the Cauchy problem (3.2) -(3.3) has a unique solution (σ, u, v, w) in C$([0,T_1]; H^N(\mathbb{R}^3))$. Suppose that $A, \tilde{A}$ are two solutions in C$([0,T_1]; H^N(\mathbb{R}^3))$ which satisfy (3.2).

Let $\tilde{\sigma} = \sigma_1(x,t) - \sigma_2(x,t)$, $\tilde{u} = u_1(x,t) - u_2(x,t)$, $\tilde{v} = v_1(x,t) - v_2(x,t)$, and $\tilde{w} = w_1(x,t) - w_2(x,t)$.

This solves

$$\partial_t\tilde{\sigma} + n_\infty\nabla\cdot\tilde{u} + n_\infty\tilde{\sigma} - \delta\Delta\tilde{\sigma} = \tilde{\sigma}\nabla\cdot u_1$$
$$-\tilde{u}\nabla\cdot\sigma_2 - (\sigma_1+\sigma_2)\tilde{\sigma}$$
$$\partial_t\tilde{u} + \lambda\tilde{u} + \frac{P'(n_\infty)}{n_\infty}\nabla\cdot\tilde{\sigma} =$$
$$-u_2\nabla\cdot\tilde{u} - \tilde{u}\nabla\cdot u_1 + \nabla\tilde{v} + \nabla\tilde{w} - \tilde{u}\nabla\sigma_2 -$$
$$\left(\frac{P'(\sigma_1+n_\infty)}{\sigma_1+n_\infty} - \frac{p'(\sigma_2+n_\infty)}{\sigma_2+n_\infty}\right)\nabla\sigma_2$$
$$+\left(\frac{P'(\sigma_1+n_\infty)}{\sigma_1+n_\infty} - \frac{P'(n_\infty)}{n_\infty}\right)\nabla\cdot\tilde{\sigma}$$
$$\partial_t\tilde{v} = \Delta\tilde{v} - k_1\tilde{v} + k_2\tilde{\sigma}$$
$$\partial_t\tilde{w} = \Delta\tilde{w} - k_3\tilde{w} + k_4\tilde{\sigma}. \quad (3.20)$$

Multiplying $\tilde{\sigma}$ to both sides of the first equation of (3.20) and integrating over $\mathbb{R}^3$, we have

$$\int_{\mathbb{R}^3}\tilde{\sigma}\,\partial_t\tilde{\sigma}dx + n_\infty\int_{\mathbb{R}^3}\tilde{\sigma}\,\nabla\cdot\tilde{u}dx + n_\infty\int_{\mathbb{R}^3}\tilde{\sigma}\,\tilde{\sigma}dx - \delta\int_{\mathbb{R}^3}\tilde{\sigma}\,\Delta\tilde{\sigma}dx = \int_{\mathbb{R}^3}\tilde{\sigma}\;\tilde{\sigma}\nabla\cdot u_1 dx - \int_{\mathbb{R}^3}\tilde{\sigma}\;\tilde{u}\nabla\cdot\sigma_2 dx - \int_{\mathbb{R}^3}\tilde{\sigma}\,(\sigma_1+\sigma_2)\tilde{\sigma}dx.$$

Using integration by parts and the Cauchy-Schwarz inequality, we get

$$\frac{1}{2}\frac{d}{dt}\|\tilde{\sigma}\|_{L^2}^2 + n_\infty\|\tilde{\sigma}\|_{L^2}^2 + \delta\|\nabla\tilde{\sigma}\|_{L^2}^2 \leq$$
$$\frac{n_\infty}{2}\|\nabla\tilde{\sigma}\|_{L^2}^2 + \frac{n_\infty}{2}\|\tilde{u}\|_{L^2}^2$$
$$+ c\|\nabla\cdot u_1\|_{L^\infty}\int_{R^3}|\tilde{\sigma}|^2dx +$$
$$c\|\nabla\sigma_2\|_{L^\infty}\int_{R^3}(|\tilde{\sigma}|^2+|\tilde{u}|^2)dx +$$
$$\|(\sigma_1)\|_{L^\infty}\int_{R^3}|\tilde{\sigma}|^2dx. \quad (3.21)$$

We establish the energy estimates for $\tilde{u}$. By multiplying $\tilde{u}$ to both sides of the second equation of (3.20) and integrating in x, we obtain

$$\int_{\mathbb{R}^3}\tilde{u}\,\partial_t\tilde{u}dx + \lambda\int_{\mathbb{R}^3}\tilde{u}\,\tilde{u}dx = -\int_{\mathbb{R}^3}\tilde{u}\,(\tilde{u}\nabla\cdot u_1)dx$$
$$-\int_{\mathbb{R}^3}\tilde{u}\,(u_2\nabla\cdot\tilde{u})dx + \int_{\mathbb{R}^3}\tilde{u}\,\nabla\tilde{v}dx - \int_{\mathbb{R}^3}\tilde{u}\,\nabla\tilde{w}dx$$
$$-\int_{\mathbb{R}^3}\tilde{u}\left(\frac{P'(\sigma_1+n_\infty)}{\sigma_1+n_\infty} - \frac{p'(\sigma_2+n_\infty)}{\sigma_2+n_\infty}\right)\nabla\sigma_2 dx$$

$$- \frac{p'(n_\infty)}{n_\infty}\int_{\mathbb{R}^3}\tilde{u}\,\nabla\,\tilde{\sigma}dx + \int_{\mathbb{R}^3}\tilde{u}(\frac{p'(\sigma_1+n_\infty)}{\sigma_1+n_\infty} - \frac{p'(n_\infty)}{n_\infty})\nabla\tilde{\sigma}dx.$$

By using integration by parts and the Cauchy-Schwarz inequality, we have

$$\frac{1}{2}\frac{d}{dt}\|\tilde{u}\|_{L^2}^2 + \lambda\|\tilde{u}\|_{L^2}^2 \le c\|\nabla\cdot u_1\|_{L^\infty}\|\tilde{u}\|_{L^2}^2 + c\|\nabla\cdot u_2\|_{L^\infty}\|\tilde{u}\|_{L^2}^2 + c\|\nabla\tilde{v}\|_{L^2}^2 + c\|\tilde{u}\|_{L^2}^2 + c\|\nabla\tilde{w}\|_{L^2}^2 + c\|\nabla\sigma_2\|_{L^\infty}\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{\sigma}\|_{L^2}^2\right) + \frac{P'(n_\infty)}{2n_\infty}\left(\|\tilde{u}\|_{L^2}^2 + \|\nabla\tilde{\sigma}\|_{L^2}^2\right) + \|\sigma_1\|_{L^\infty}\left(\|\tilde{u}\|_{L^2}^2 + \|\nabla\tilde{\sigma}\|_{L^2}^2\right).$$

Since L^∞ norms of σ^i, u^i, v^i, w^i, where i = 1, 2 are bounded, we have

$$\frac{1}{2}\frac{d}{dt}\|\tilde{u}\|_{L^2}^2 + c_1\|\tilde{u}\|_{L^2}^2 \le c\|\nabla\tilde{v}\|_{L^2}^2 + c\|\nabla\tilde{w}\|_{L^2}^2 + c\|\nabla\tilde{\sigma}\|_{L^2}^2 + c\|\tilde{\sigma}\|_{L^2}^2. \qquad (3.22)$$

We have a similar way to estimate $\tilde{v}$ and $\tilde{w}$ as follows:

$$\frac{1}{2}\frac{d}{dt}\|\tilde{v}\|_{L^2}^2 + \|\nabla\tilde{v}\|_{L^2}^2 + k_1\|\tilde{v}\|_{L^2}^2 \le k_2\left(\|\tilde{v}\|_{L^2}^2 + \|\tilde{\sigma}\|_{L^2}^2\right) \qquad (4.23)$$

$$\frac{1}{2}\frac{d}{dt}\|\tilde{w}\|_{L^2}^2 + \|\nabla\tilde{w}\|_{L^2}^2 + k_3\|\tilde{w}\|_{L^2}^2 \le k_4(\|\tilde{w}\|_{L^2}^2 + \|\tilde{\sigma}\|_{L^2}^2). \qquad (3.24)$$

Taking the linear combination of all estimates, we get

$$\frac{1}{2}\frac{d}{dt}\left(\|\tilde{\sigma}\|_{L^2}^2 + \|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2 + \|\tilde{w}\|_{L^2}^2\right) + c_1((\|\tilde{\sigma}\|_{L^2}^2 + \|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2 + \|\tilde{w}\|_{L^2}^2) + c_2((\|\nabla\tilde{\sigma}\|_{L^2}^2 + \|\nabla\tilde{w}\|_{L^2}^2 + \|\nabla\tilde{v}\|_{L^2}^2) \le c\left(\|\tilde{\sigma}\|_{L^2}^2 + \|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2 + \|\tilde{w}\|_{L^2}^2\right). \qquad (3.25)$$

Applying Gronwall's inequality to the above inequality, one has

$$\sup_{0\le t\le T_1}(\|\tilde{\sigma}\|_{L^2}^2 + \|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2 + \|\tilde{w}\|_{L^2}^2) \le e^{\int_0^t c ds}(\|\tilde{\sigma}(0)\|_{L^2}^2 + \|\tilde{u}(0)\|_{L^2}^2 + \|\tilde{v}(0)\|_{L^2}^2 + \|\tilde{w}(0)\|_{L^2}^2).$$

Since the initial data of $(\tilde{\sigma}, \tilde{u}, \tilde{v}, \tilde{w})$ are all zero for T > 0, that implies the uniqueness of the local solution.

4. Conclusion

In this paper, we prove the existence of local solutions for an attraction-repulsion chemotaxis fluid model with logistic sources in three dimensions. We show the existence of local solutions by the energy method. We divided the proof into four steps, using integral by parts, Cauchy –Schwarz inequality, and Gronwall's inequality to prove these steps.

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